

String Theory: A Theory of Unification

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Abstract

These notes on string theory are based on a series of talks I gave during my graduate studies. As the talks, this introductory essay is intended for young students and non-string theory physicists.

1 Introduction

For more than twenty years the standard model of particle physics has successfully described data from experiments probing energies up to the order of 100 GeV. It has also predicted new phenomena and particles subsequently detected, and it continues to provide us with new insights into the microscopic physical world. In spite of the fact that the standard model is the most reliable theory we have to describe phenomena at subnuclear scales, at higher energies new physics is expected to take place and new theoretical frameworks are required. A viable model is string theory, a theory describing physics at very such high energies that quantum gravitational effects cannot be neglected (Planck scale $\sim 10^{19}$ GeV). It is worth stressing that in general a new high energy description do not need to be a simple extension of the standard model, in fact, even if the former should reduce to it in the limit of low energies, it can be different at the conceptual level. This is the case of string theory, where new concepts such as worldsheet, branes, extra dimensions and so on are introduced. In this essay I shall focus on string theory, a possible quantum theory of gravity and moreover expected to be a unified model of all the fundamental interactions. In the last part I comment on a novel approach relating confining gauge theories and string theory: the AdS/CFT correspondence.

2 Symmetries and Unification

As in many other branches of physics, symmetries also play a crucial role in the search for a unified theory of nature. In particular, a new symmetry relating bosons and fermions has emerged as a fundamental ingredient. This symmetry is called supersymmetry (SUSY) [1] and has changed our understanding of the spacetime in a dramatic way. Indeed, SUSY is the only possible extension of

the four-dimensional Poincaré symmetry, and it is built adding anticommuting directions to the usual spacetime. The spectrum of the supersymmetric extension of the Poincaré algebra indicates that every boson has a fermionic supersymmetric partner and vice versa. At low energies there is no experimental evidence for supersymmetric particles, nevertheless, at high energies it is hoped that SUSY becomes an exact symmetry.

In addition to this, SUSY is considered a fundamental principle of any unified theory. In a unified description of all the fundamental interactions it is required that the coupling constant of each force, the parameter characterizing its strength, coincides with all the others at the energy of unification. But, the couplings of the standard model in general depend on the particle content and it is known that they never coincide in a common value, nevertheless, including supersymmetry the spectrum gets modified in such a way that it provides a better possibility for unification ($E \sim 10^{17}$ GeV).

The formulation of a viable grand unified theory (GUT) is one of the main motivations for studying SUSY, another reason is that SUSY gives valuable hints in order to solve the related hierarchy problem. The latter is a non-trivial puzzle of the standard model and it concerns the existence of two very different energy scales in the model. One of these is the electroweak scale (~ 100 GeV) and the other one is a very high energy supposed to be the Planck scale. The presence of such huge energy is theoretically suggested in the low energy dynamics by the well known divergent radiative corrections to scalar masses¹. Hence, to solve the hierarchy problem the divergences of the standard model should be smoothed. Introducing SUSY this is achieved since an equal number of propagating bosonic and fermionic degrees of freedom cancel each other in loop diagrams. With this simple artifact SUSY solves the technical aspects of the hierarchy problem.

Another related long standing problem where SUSY has also played an important role is in the reconciliation of general relativity and quantum field theory, that is, in the formulation of a quantized theory of gravity. An elegant scheme for this is provided by supergravity (SUGRA) [2], a model where supersymmetry is defined locally. It turns out that the spectrum of the theory contains a particle whose dynamics is dictated by Einstein equations, so such particle is naturally identified with the graviton. However, the introduction of SUSY does not get rid of all the infinities this quantized theory of gravity shows. Actually, at short distances it can be shown that renormalizability becomes useless. This is a reliable sign that SUGRA, local super-Poincaré, is also a low energy regime of a more fundamental theory. Nowadays, a large amount of theoretical evidences converge to indicate that this formulation is realized by string theory.

3 String Theory

String theory [3] was originally proposed as a model, the old dual string model, for describing strong nuclear interactions. However, it was discarded

¹This suggests that the standard model must be thought of as an effective field theory of a more fundamental theory incorporating gravity.

due to the success of quantum chromodynamics [4], the latter accounting satisfactorily for the great number of data arising at the time from high energy experiments (deep inelastic scattering). At that time, string theory suffered from two main physical deficiencies: first, the presence in its spectrum of a massless spin-2 particle not revealed in experiments and, secondly, its mathematical consistency only in spacetime dimensions larger than four.

The first issue was solved noting that the presence of a massless spin-2 particle could indeed be evidence that the string model was something more than a simple theory of the strong nuclear force, in fact, after identifying the spin-2 particle with the graviton, the model was elevated to a quantized theory of gravity. Furthermore, very remarkably, it was shown that the low energy limit of string theory coincided with some already known SUGRA, where it is known that at lower energies the graviton interacts as in general relativity.

The trouble caused by the extra dimensions was overcome borrowing an old idea from Kaluza and Klein, *i.e.*, the extra dimensions are thought to be so small, more or less of the same size than the string length, that they cannot be perceived in daily life experience. More recently it was conjectured that the extra dimensions could even attain the TeV scale. These new models are collectively called brane-worlds and it is eagerly expected that they will give rise to testable results in high energy physics or even in cosmology.

The crucial difference between strings and point particles is that strings have an infinite internal number of degrees of freedom, they can vibrate, while point particles can only propagate in space. This freedom to vibrate, with each oscillation associated with a particle, gives rise to a characteristic spectrum. In addition to the ten-dimensional graviton, the spectrum also contains the dilaton field ϕ whose vacuum expectation value determines the unique string coupling constant $g_s = e^{\langle\phi\rangle}$ and allows for a perturbative analysis of the theory.

In contrast to supergravity string theory is free of ultraviolet divergences. Let us see how this works. Generalizing quantum field theory techniques, a perturbative formulation of string theory is carried out summing over all possible histories of stringy Feynman topological diagrams, each of them with certain power of the coupling. For example, each genus h Riemann surface representing an interacting level of closed strings comes with a factor g_s^{2h+2} with $h = 0, 1, 2, \dots$. Proving that each of these stringy diagrams is ultraviolet finite the theory overcomes the problem with short distance singularities of supergravity and other point particle models.

In addition to the one-dimensional fundamental string, superstring theories also include higher dimensional objects called Dp -branes. These objects are p -dimensional black hole type solutions of SUGRA (see below) and are the natural higher dimensional generalization of point particles ($p = 0$) and strings ($p = 1$). Moreover, like point particles can be electrically charged, Dp -branes can carry Ramond-Ramond charges. The D is for Dirichlet and indicates that open strings have their end points living on the brane. Therefore, open strings are constrained to move with their ends attached to these hyperplanes. On the other hand, closed strings can move freely in all the space. This picture suggests that the only manner an open string can get away from the brane is if both ends join at some point, form a closed string and then escape far away. Since open string excitations are associated with gauge fields living on

the branes, whereas gravitational fields are mediated by closed strings, whose lines of force can invade all the dimensions, we are lead to think that there is a closed connection between particle and gravitational physics.

4 Nuclear Forces and Strings

At high energies non-Abelian gauge theories as QCD are weakly coupled, opening the possibility for exact predictions. On the other hand, in the infrared the theory is strongly coupled and a perturbative analysis is nonsense. The strong behavior QCD shows at low energies explain why quarks, the constituents of nucleons, cannot be isolated in experiments and remain coupled to each others forming different composite particles. Confinement is an important unsolved problem in particle physics that string theory has contributed to clarify.

The old dual string model supposed that the chromo-electric fields emitted by quarks, and binding them together, were thin tubes or strings (the ‘QCD-string’). In this picture confinement finds a natural explanation since farer the quarks are brought apart greater is the energy needed to continue separating them. Nevertheless, strictly speaking QCD is not a confining theory since at energies beyond the QCD scale $\Lambda_{QCD} \sim 300$ MeV the vacuum starts to create quark-antiquark pairs and thus the original quarks can be separated as much as desired. Since this last phenomenon is still more complex than pure confinement theorists have concentrated mostly on the latter.

QCD-like theories, including their supersymmetric extensions, are real confining theories as long as they satisfy at least one of the two following conditions: all the quarks are much heavier than the QCD scale, the point where the vacuum breaks, or the number of family of quarks is very large. The importance of these variants is that confinement survives these limits and many simplifications occur, allowing for new theoretical insights of the real problem.

The introduction of a large number of colors, $N \rightarrow \infty$, changes the expansion parameter of the original Yang-Mills theory g_{YM} to a new effective ’t Hooft Coupling $\lambda \equiv g_{YM}^2 N$. The new expansion parameter now allows for a Feynman-like diagrammatic in Riemann surfaces with increasing genus. This is an old result and it is striking how similar it is to the perturbative expansion of string theory we saw above, nevertheless, only recently it was shown that large N gauge theories and string quantum gravity have indeed an equivalent (dual) mathematical description.

5 String / Gauge Theory Duality

Maxwell equations with magnetic monopoles are invariant under the interchange of electric and magnetic quantities. The two theories are said to be dual. The classical comparison of these theories is not interesting since they are completely analogous, nevertheless, what is relevant is that the dual quantized versions of them are quite different. What in a first moment was a perturbative analysis in terms of the weak electric coupling, in the dual magnetic description becomes a non-perturbative strongly coupled system. The D-branes mentioned

above, with their tension going as g_s^{-1} , are the analogue in string theory of these dual magnetic objects. In both cases they help to probe the physics at strong coupling. The AdS/CFT conjecture we shall study below makes use of this idea, now proposing a duality between a gravity and a non-Abelian gauge theory [5].

In order to clarify this duality it is convenient to introduce the concept of black hole, a very massive object originated in a gravitational collapse, inside of which all the forces of nature are in action. For our purposes, a black hole [6] simply can be regarded as a region of spacetime from where no information can escape beyond its boundary, *i.e.*, the information inside the black hole is inaccessible to distant observers. Moreover, black holes are very simple objects since their properties do not depend on the kinds of constituents they are made of, but instead on some basic properties such as mass, charge and angular momentum.

The Schwarzschild black hole is the simplest one and it has a event horizon which is a sphere of area $A = 4\pi G^2 M^2 / c^4$. It can be systematically proved that this area cannot decrease in any classical process. On the other hand, gravitational collapsing objects which give rise to black holes seem to violate the second law of thermodynamics. This is easy to see since the initial collapsing object has a non-vanishing entropy whereas the final black hole cannot radiate, then the entropy of the entire system has decreased. The problem is solved by providing an entropy to the black hole. For a Schwarzschild black hole it was proposed by Bekenstein that the entropy is proportional to the event horizon area, a quantity that can only increase as the entropy does in classical thermodynamics,

$$S_{BH} = \frac{1}{4} \frac{A}{l_p^2} . \quad (1)$$

The generalized second law of thermodynamics extends the usual second law to include the entropy of black holes in a composite system, counting the entropy of the standard matter system and also that of the black hole $S_{TOT} = S_{MATT} + S_{RAD} + S_{BH}$. This is the entropy that always increases. Starting with a collapsing object of entropy S , the generalized second law of thermodynamics imposes that $S \leq S_{BH}$. This is the holographic bound, and it states that the entropy of a matter system entirely contained inside a surface of area A , cannot exceed that of a black hole of the same size. Alternatively, the holographic bound can be rephrased saying that the information of a system is completely stored in its boundary surface.

This statement is generalized by the holographic principle [7]. It claims that any physical process occurring in $D + 1$ spacetime dimensions, as described by a quantum theory of gravity, can be equivalently described by another theory, without gravity, defined on its D -dimensional boundary. Some authors believe that this statement is universal and a fundamental principle of nature. Nevertheless, the principle has been tested only in a few concrete cases. An exception is the AdS/CFT correspondence, since it exactly relates superstrings in a D -dimensional space with a superconformal field theory on the boundary.

Finally, let us comment on the black hole information paradox and see how the holographic principle resolves it. The paradox can be posed in the following terms. If the initial collapsing object is in a pure quantum state

before it starts to contract, we expect the final object to be in exactly the same configuration. However, the thermal radiation of the final object comes necessarily as mixed states and so the information we get about the inside does not reproduce the information booked in the original object. We can say that the initial information is lost or destroyed inside the black hole. This paradox is solved by the holographic principle since the full dynamics of the gravitational theory is now described by a standard, though complex, quantum system with unitary evolution.

So far the most accurate holographic proposal relating gauge theories to a quantized theory of gravity is the novel AdS/CFT correspondence. In two words, it says that string theory defined in a negatively curved anti-de Sitter space (AdS) is equivalent to a certain conformal field theory (CFT) living on its boundary. One concrete example is AdS₅/CFT₄: it states that type IIB superstring theory in AdS₅ is equivalently described by an extended $\mathcal{N} = 4$ super-CFT in four dimensions. The other five dimensions of the bulk are compactified on S⁵. The five-sphere with isometry group $SO(6)$ is chosen in order to match with the $SU(4)$ R-symmetry of the super Yang-Mills theory.

The AdS₅/CFT₄ correspondence can be motivated as follows. Starting with N parallel D3-branes on top of each other, we have open strings ending on the branes and closed strings moving in the whole space. On the branes the open string dynamics is that of a weakly coupled gauge theory. Since Dp -branes are black holes the latter description should coincide with that of strongly interacting closed strings near the horizon. Hence, the strongly coupled gravitational theory in the bulk is equivalent to a weakly coupled gauge theory on the brane, and viceversa. This is the basic statement of the AdS/CFT conjecture.

Since the AdS/CFT correspondence is a weak/strong duality it allows us to probe the strong coupling regime of the gauge theory with a perturbative analysis in the string side. This statement is established by the fundamental relation between the two sides of the correspondence

$$\left(\frac{R}{l_s}\right)^4 = g_s N \leftrightarrow g_{YM}^2 N \equiv \lambda, \quad (2)$$

where R is the curvature radius of the anti-de Sitter space, N is the number of colors (in the large limit $N \rightarrow \infty$) in the gauge group and λ is the 't Hooft coupling.

In the supergravity limit the string length is much smaller than the radius of the AdS space, given

$$1 \ll \left(\frac{R}{l_s}\right)^4 \leftrightarrow \lambda. \quad (3)$$

In this limit the bulk theory is manageable, being a gauged supergravity theory, but in the boundary side it turns out that the gauge theory is in a strongly coupled regime, where a perturbative analysis is senseless. This establishes the weak/strong coupling nature of the duality. This is a clear advantage as we want to study the strongly coupled regime of one of the theories, since we can always use perturbative results in the dual theory. However, the difficulties in finding a common perturbative sector where to test the correspondence makes it hard to

prove its full validity. The strong formulation of the AdS/CFT correspondence claims its validity at the string quantum level, nevertheless, so far nobody has been able to quantitatively prove it beyond the supergravity approximation.

The main challenges of AdS/CFT are two-fold: i) to shed light in the strongly coupled regime of non-Abelian gauge theories, as a step further in the understanding of more realistic QCD-like theories; ii) to provide a full proof of the correspondence. The latter is a non-trivial task since we do not have an independent non-perturbative definition of string theory that could be compared with the boundary theory at the strong regime. Pointing in this direction, a couple of years ago a new proposal was suggested, that goes under the name of BMN conjecture [8, 9], and opened the possibility to test the correspondence beyond the SUGRA limit. The idea is to investigate the consequences certain limiting procedure, namely, the Penrose Limit, has on both sides of the correspondence. In this limit not all the ideas involved are conceptually well established. One of these is the fate of holography in the BMN limit. It seems that the beautiful holographic picture that the AdS/CFT duality shows is completely lost in the plane-wave background.

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